Corpus Christi College Estimation Evening: Answers

M. Sutherland, November 2 2010

There are no unique right or wrong ways of answering these questions. The following solutions are merely meant to give an example of how one might go about estimating the quantities asked for.

1. *The 'London Eye' is a large Ferris wheel that is a popular London tourist attraction. ck's law What is the rotational kinetic energy of the London Eye when it is fully loaded and moving at normal speed?* [answer in J]

   Answer: We can use the formula KE = $1/2\ I\omega^2$ where I is the moment of inertia and ω is the angular velocity. To do this we need to estimate the mass, radius and speed of rotation of the wheel. Let's start with mass. There are about 50 capsules, each of mass say 10 000 kg. Then there is the structural steel and supports, which probably weigh at least that much again. We are ignoring the mass of the people (convince yourself that this is a good approximation). I then get m = 50 × 2 × 10^4 kg = 10^6 kg.

   I estimate the eye stands about 100 m tall, so r=50 m. To calculate I we use the fact I=mr^2 for a ring. It moves pretty slowly, I would guess about a revolution per 30 minutes so ω= 3 × 10^{-3} rad s^{-1}. So I get

   KE = (1/2)(10^6kg)(50 m)^2(3 × 10^{-3} rad s^{-1})^2 = 15 000 J. Using the correct data from the engineering specifications yields something quite close, KE = 5 × 10^4 J.

2. *How many physics papers are published worldwide each year?* [answer in number of papers]

   Answer: 'Too many' would be a good answer. However if we want to be more quantitative we could estimate there are about 50 countries doing active physics research, each with roughly 30 departments consisting of 10 research groups. If each of these groups publishes 10 papers per year we get about 1.5 × 10^5 papers.

   A quick check of the ISI citation index indicates there were actually 1.4 × 10^5 papers published in all categories of physics in 2009.

3. *On a clear night, roughly how many visible light photons per second make it to your eye from Alpha Centauri A (a sun-like star about 4 light years away?)* [answer in photons s^{-1}]

   Answer: A sunlike star radiates about 1.4 × 10^3 W m^{-2} at the orbital distance of the Earth. How much of this lies in the visible range of the spectrum? A tough quantity to estimate. We could think of the star emitting a black body-like spectrum which is likely peaked somewhere in the visible region of the spectrum (since plants and our eyes evolved to use visible light and nature is efficient!). It probably isn't too bad an approximation then to claim that most of the power radiated by a sun-like star falls in the visible range. We know that the irradiance drops off as $1/r^2$, so the by the time the light from Alpha Centauri A travels to our eye we would have:

$$I = 1 \times 10^3 W m^{-2} \left(\frac{8lm}{2 \times 10^6 lm} \right)^2 = 2 \times 10^{-8} W m^{-2} \tag{1}$$

   My eye is about 2 cm in diameter, so an area of 10^{-3} m^2. If we take a typical photon wavelength to be 600 nm then the energy of this photon is E=hc/λ = 3×10^{-19} J. Thus, the number of photons arriving per second is 2 ×10^{-8} W m^{-2}/(3×10^{-19} J) × 10^{-3} m^2 = 7 × 10^7 photons s^{-1}.

4. *Last month astronomers announced the detection of Glise 581g†, which is believed to be the first extrasolar planet observed that possesses an Earth-like mass and range of temperatures. The scientists inferred its presence by observing the changes in the radial velocity of the star Glise 581 via the Doppler shift, as it and the planet orbit about a common center of mass. The Glise planetary system is thought to be almost coplanar to our own. If aliens on Glise 581g were to detect Earth using the same method, what is the required resolution of their instrument?* [answer in m s^{-1}]

   Answer: The derivation of the radial velocity equation is tedious. Let's try a simpler way. We know the Sun and Earth will revolve about a center of mass. Where is this located?

$$COM = \frac{\Sigma m_i r_i}{\Sigma m_i} \tag{2}$$

   We can take the Sun to be at r=0 and the Earth - Sun distance to be 8 light minutes. The only tricky bit is estimating the mass of the sun, which can be done a number of ways. It turns out to be about 330,000 times more massive that the Earth. Isaac Newton got a figure of about half of this when he wrote his Principia. Putting all this together we find that the Earth-Sun system revolves around a point 4×10^5 m from the center of the sun, with a period of 1 year.

   Assuming a circular orbit, this means that the velocity of the Sun as it moves about the COM is $(2\pi r)/(1 \text{ year})$ = 0.1 m s^{-1}. Aliens on Glise would see the radial velocity oscillate between $\pm$ 0.1 m s^{-1} over the course of a year, so would need to build an instrument with at least this resolution. Note that the best detectors on Earth have a resolution about 10 times worse than this, but are improving rapidly!

5. *What is the energy cost of removing 1 kg of CO_2 from the atmosphere?* [answer in J]

   Answer: We can come up with an estimate by considering the change in the Gibb's free energy for mixing two gases. For n moles of a substance, moving from an ambient partial pressure P_0 to a final partial pressure P, the energy change is:

$$\Delta G = nRTln\frac{P}{P_0} \tag{3}$$

   1 kg of CO_2 is about 22 moles. The current concentration of CO_2 in the atmosphere is about 400 ppm, and we would probably like to extract this to a final partial pressure of 1 bar, say for sequestration underground. Using these values, and the equation above, gives us an estimate of 4×10^5 J.

6. *What is the pressure at the center of the Earth?* [answer in Pa]

   Answer: We can calculate the pressure by knowing that g goes linearly between 9.8 m s^{-2} at the surface of the Earth and 0 at the center. We can estimate a density of 5×10^3 kg m^{-3}, which you can work out from the radius and mass of the Earth. After a bit of algebra you can show that the pressure at the center of the Earth is give by:

$$P = \frac{g(R_E)\rho}{R_E}\int_0^{R_E} rdr \tag{4}$$

   This gives us P = (10 m s^{-2})(1/2)(5×10^3 kg m^{-3})(6×10^6 m) = 1.5×10^{11} Pa. This is quite close to the value that geophysicists agree on, 3.6×10^{11} Pa.